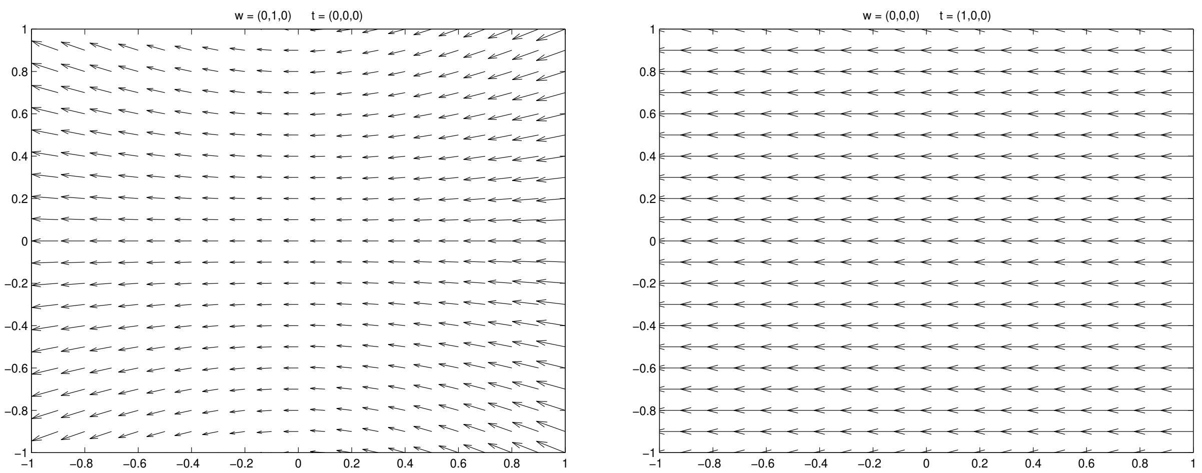
- Rotation v/s translation ambiguity
- Stereopsis: description, vergence and version movements, disparity

1 Rotation v/s translation ambiguity

Figure 1 shows the optical flow produced by a rotation around Y axis and a translation along X axis. It can be observed that near the origin both flows are pretty similar, which difficults the task of estimating the motion, specially in the presence of noise. Errors in motion estimation would also affect depth estimation. For example, if we overestimate the rotational part, depth estimation will be flat and if we overestimate the translational part, depth will be cute. This is because depth affects only the translational part of the optical flow. To avoid this problem (called **Bas-Relief Ambiguity**) it is necessary to have a large field of view (FOV).



(a) Rotation around Y axis (b) Translation along X axis

Figure 1: Bas-Relief Ambiguity

2 Stereopsis

2.1 Description

Consider two cameras LE (left eye) and RE (right eye), with fixed optical centers OL and OR, located at a distance b (See Figure 2). Each camera has 3 degrees of freedom (d.o.f.)

corresponding to rotation around the 3 axes. In the case of human beings, these 6 d.o.f. are restricted by the following:

- **Listing's law**: the axis of rotation of the eye lies on the front parallel plane, i.e., rotation around the optical axis (Z axis) is restricted.
- **Fixating system**: the eyes move in such a way that both point to the same arbitrary point in the space, namely fixation point.

These two restrictions reduce the d.o.f. of the stereo vision system to 3.

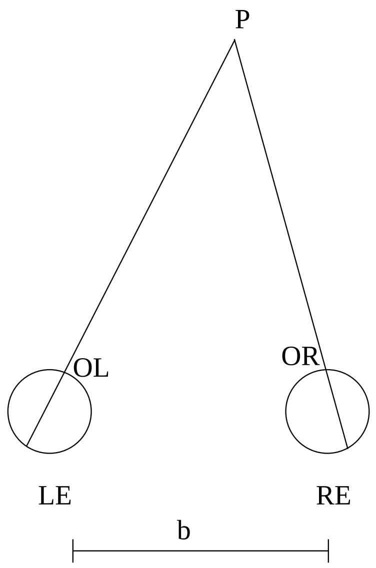
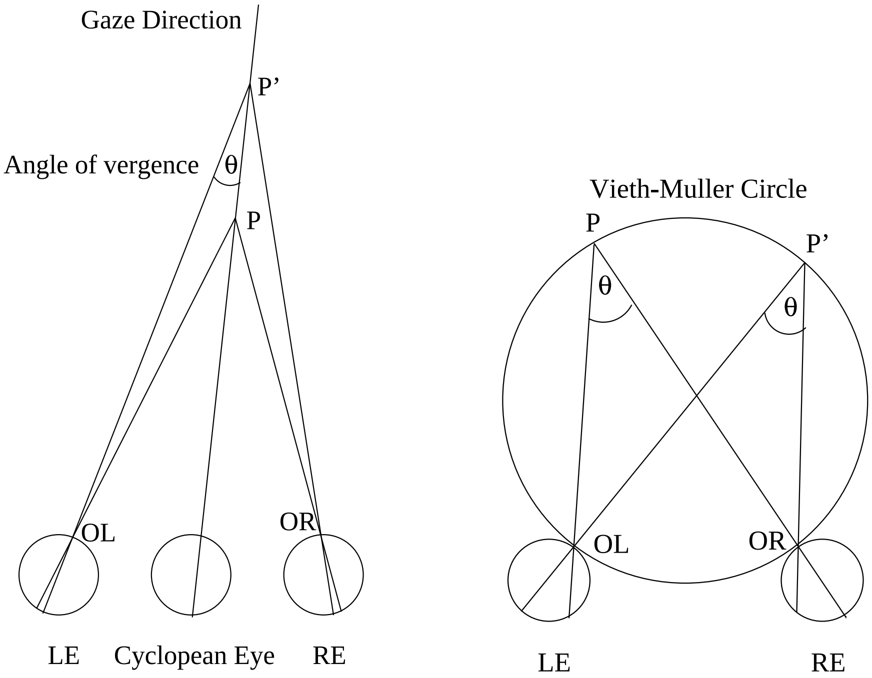


Figure 2: Stereopsis

2.2 Vergence and version movements

For the time being, restrict ourselves to fixation points located in a horizontal plane. In this case we can classify the motion of the eyes in two different movements (See Figure 3):

- **Vergence movement**: it is a movement in which both eyes rotate in the same angle, but in opposite directions. In this case the **angle of vergence** changes while the fixation point moves along the **gaze direction**. Note that if the angle of vergence is not too small, it can be used to measure depth.
- **Version movement**: it is a movement in which gaze direction changes, but angle of vergence remains constant. From Euclidean geometry we now that this happens when the fixation point lies on a circle. This circle is called **Viet-Muller Circle**.



(a) Vergence movement

(b) Version movement

Figure 3: Vergence and version movements

2.3 Disparity

Consider that the stereo system fixated at a point P, whose projections in LE and RE are P_L and P_R, respectively (See Figure 4). Now look at a point P' with projections P_L' and P_R'. Define $\theta_L = \angle P_L O_L P_L'$ and $\theta_R = \angle P_R O_R P_R'$. Then the **disparity** is defined as $\theta_R - \theta_L$. Note that the disparity remains constant if the angle of vergence is fixed.

2.3.1 Example

Consider the configuration of Figure 5. For θ small we have

$$
\begin{aligned}
Disparity &= 2\delta\theta \\
\theta &\approx \tan\theta = \frac{b}{2z} \\
\theta - \delta\theta &\approx \frac{b}{2(z+\delta z)} \\
\delta\theta &\approx \frac{b}{2z} - \frac{b}{2(z+\delta z)} \\
Disparity &\approx \frac{b\delta z}{z^2}
\end{aligned}
$$

In the case of human beings, best stereo accuracy is obtained for a disparity of approximately 5"$= 2.42 \times 10^{-5}$ radians.

For $b = 6$ cm, $z = 100$ cm, $\delta z = 2.42 \times 10^{-5} \times 100^2/6 = 0.04$ cm (good precision).

For $b = 6$ cm, $z = 1000$ mt, $\delta z = 2.42 \times 10^{-5} \times 1000^2/6 = 404$ mt (bad precision).

Therefore a stereo system works well for short distances.

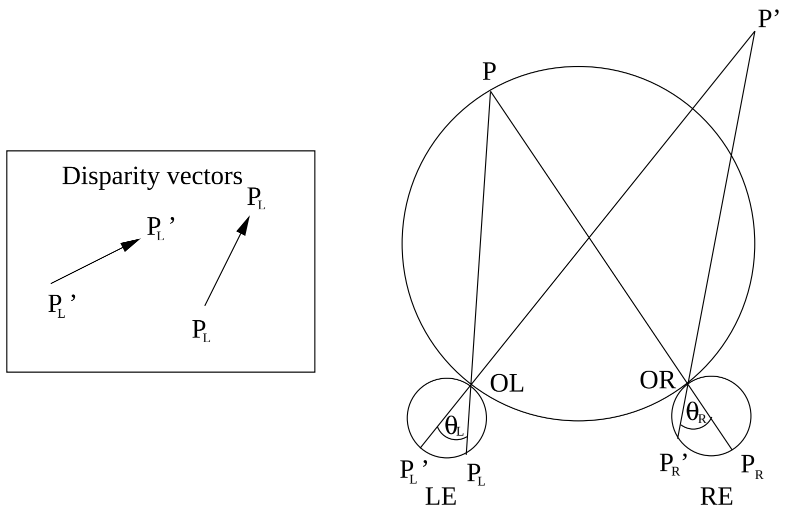


Figure 4: Disparity

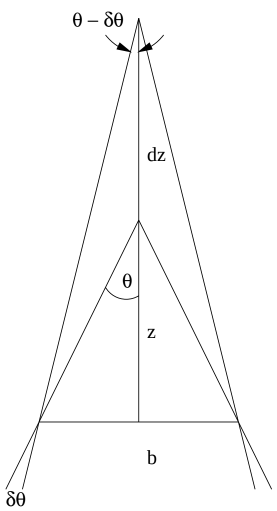


Figure 5: Example