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Lecture Notes
CS294 Visual Grouping and Recognition
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# Good Continuation of Boundary Contour

## Review

Factors that leads to grouping.

- ✓ Similarity (brightness, color , texture, disparity, motion, …) Done!
- ✓ Proximity (location) Done!
- Good continuation of boundary contour (today's lecture)
- Closure (not yet covered)
- Symmetry and parallelism (not yet covered)
- Familiar configuration (important since most things in nature are symmetric) (not yet covered)

Remind the goals for the class: 3 things: **Grouping, figure ground, recognition**.

**Good continuation**

We want A' associates with A, and B ' associates with B since B  and B' have good continuation, and so are A and A'.

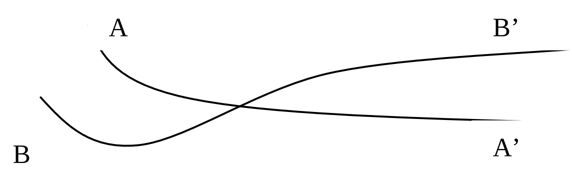



Figure. 1

Important: Need to enforce the continuity of the orientation.
Segment 1 associates with segnment 2 since the orientation from segment 1 to segment 2 doesn't change.

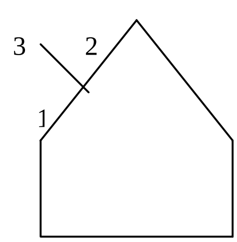


Figure. 2

**Criteria for good continuation:**
Smooth and short continuation is more likely to result in correct partition.

**Definition of Elastica:** (Due to Euler)
The curve of minimum energy which meets the boundary constraints

$$E = \int_{curve} K^2 \, ds$$

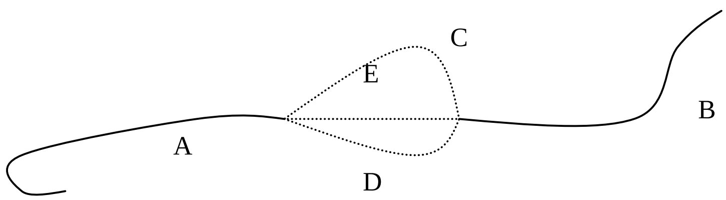



Figure. 3

Where E is the energy, K is the curvature (double derivative), and curve is the set of all possible curves that meet the two end points. As seen in figure. 3, E is the curve that as minimum energy.

We can translate the elastica into probability using

$$P = e^{(\text{-Elastica Energy/normalize constant})}$$

**Discussion on some special cases:**

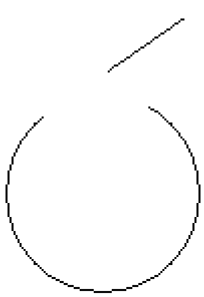


Figure. 4

Look at figure. 4, should we complete the circle or connect the circle with the tangent? The solution from minimization of energy would say one should connect the circle with the tangent, but clearly the argument for completing the circle is as strong. So, the method of minimization the energy cannot work all the time. One should use this method in conjunction with other methods for color, texture, …

The formal method for solving the energy minimization is calculus of variation. However, this method is generally very expensive. Instead, there is an approximation P' of elastica method. Note that this approximation is somewhat "black magic."

$$P' = e^{-(R/\sigma 1 - D/\sigma 2)}$$

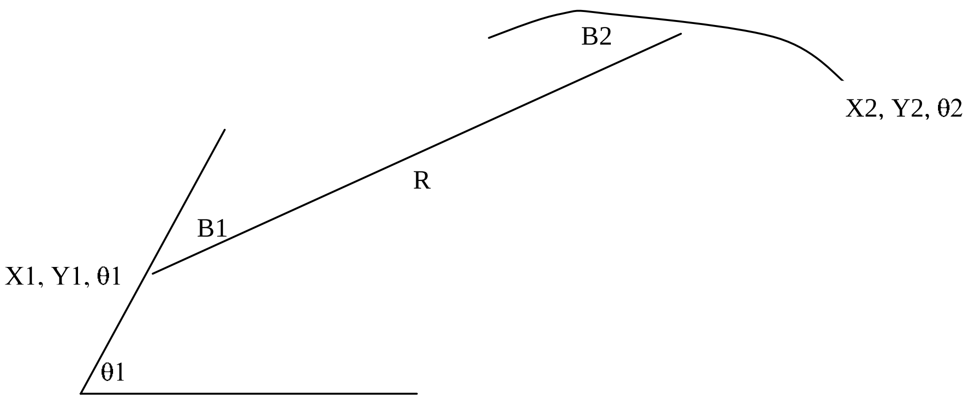



Figure. 5

$$D = B1^2 + B2^2 - B1*B2;$$

In figure 5, R is the distance between the two midpoints of the two curves. B1 and B2 are the angels made by R and the tangents to the midpoints. σ1, σ2 are the scale factors.

Now, we have the connection probability between two segments. Next, we want to select the optimal path that connects them. Suppose we pick a point P between the two curves that we want to connect. We want to find out the probability that the point is lied on the minimization energy line that connects the two curves A and B. See figure

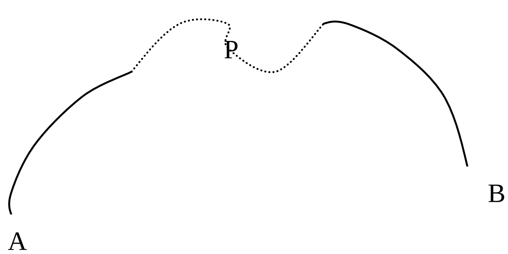



Figure. 6

1. At point P, compute the elastica of A and P for every direction (tangent) of P
2. Pick the smallest elastica AP.
3. Compute the elastica of B and P for every direction (tangent) of P
4. Pick the smallest elastica BP.

The probablity that the point P is on the line that connects A and B is

$$P = e^{-(AP+BP)}$$

**Interesting Discussion:**
Is the figure. 7 useful in computer vision at all? Does it happen in real life? Or is it just an interesting phenomena devised by the psychologist?

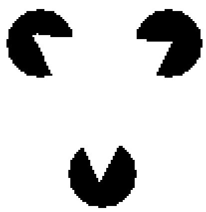
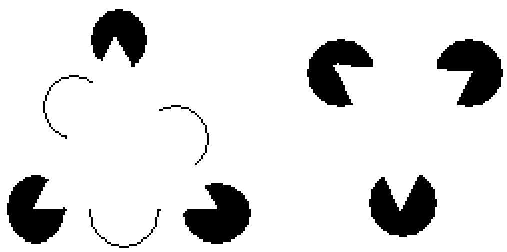


Figure. 7

The answer is yes, it is useful in practice. These situations occur quite often in computer vision. The reason is that sometimes the contrast between the object and the environment at some boundaries are so low that the object and the environment tend to blend into each other. However, if we can separate the object and the environment at other boundaries, we can eliminate this problem.

**Types of curvilinear completion**

There are two types of curvilinear completion.
Parallel to visible contour fragments
Perpendicular to the terminators



(a) (b)

Figure. 8

Figure. 8 (a) contains both perpendicular to the terminators (ortho grouping) and parallel to the visible contour fragment (para grouping). While figure. 8(b) contain only para grouping.
Note that there are two kinds of completion. One is “MODAL” completion, the other is “AMODAL” completion. “MODAL” completion tries to group the object in front while “AMODAL” completion tries to group the object in the back. For example: In the figure, “MODAL” completion would try to identify the white triangle while “AMODAL” completion would try to identify the black circles.

Clearly, ortho grouping doesn’t need to be called ortho grouping since the contours are not necessary perpendicular to the terminators. So, why do we call “ortho grouping”. Here is the reason.

Consider the random segments of lines in the figure 9. There seems to be a white object laying on top of those segment. Now what is the distribution of the angle the line created with the white object. First of all, the distribution is not uniform because at 180 degree, there is no intersection of the line with the white object at all. The distribution is also should be symmetric around 90 degree due to symmetry to distribution of the angles. At 90 degree, we would expect the probability for an intersection is highest assuming that the distribution of the segment’s length is independent with the angle. Hence, by Baysian philosophy, if given only one line (terminator) the best prediction of the contour will be the one that is perpendicular to the terminator. Hence, “ortho grouping” is appropriate.

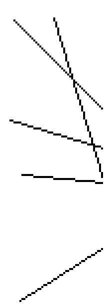
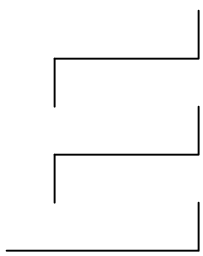
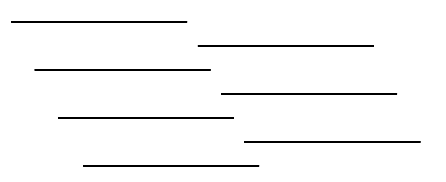


Figure. 9

Interesting phenomenas.





(a) (b)

Figure 10

Figure 10 (a) seems to be an “E”. Hence, it supports the ortho grouping view, while in figure. 10(b), there seem to be an imaginary line between the line segments, yet the terminators are not perpendicular to the contour.