

Cuts with Constraints

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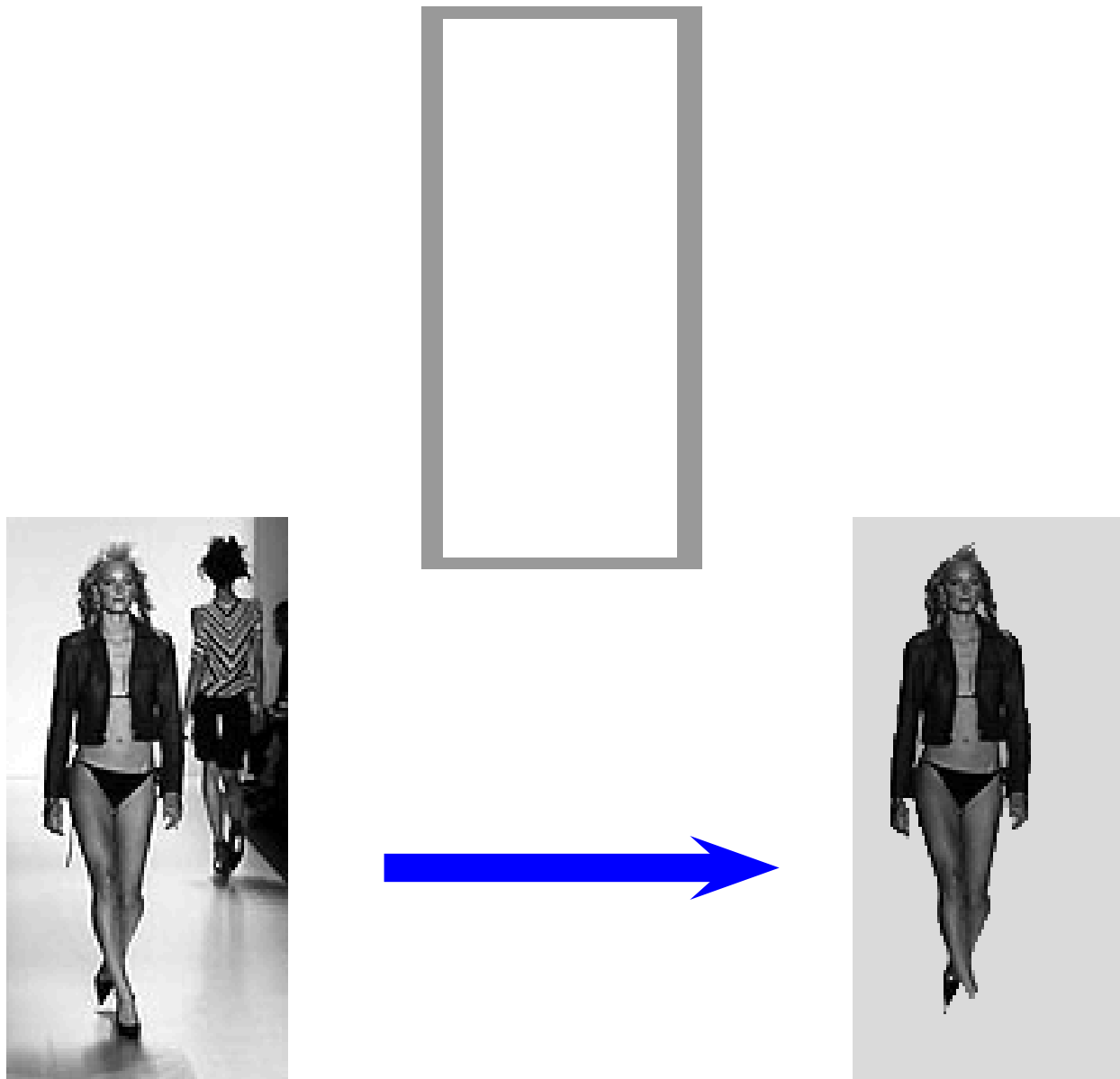
Segmentation given partial grouping



Segmentation given partial grouping



Segmentation given partial grouping



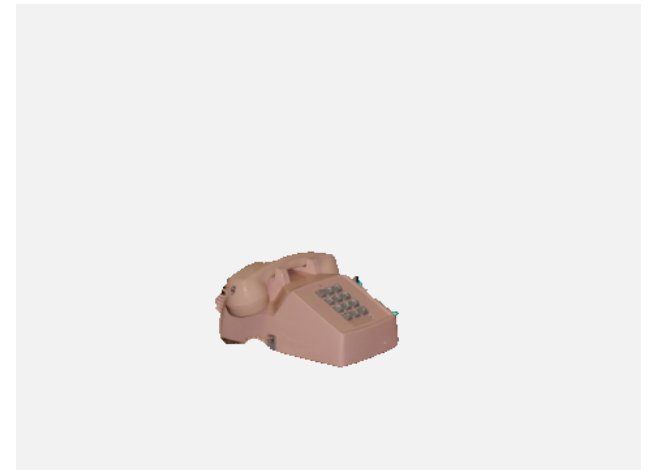
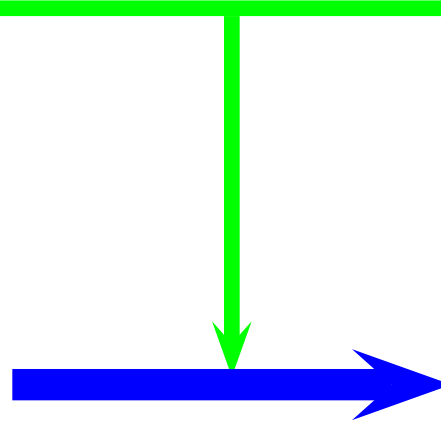
Segmentation with object knowledge



Segmentation with object knowledge



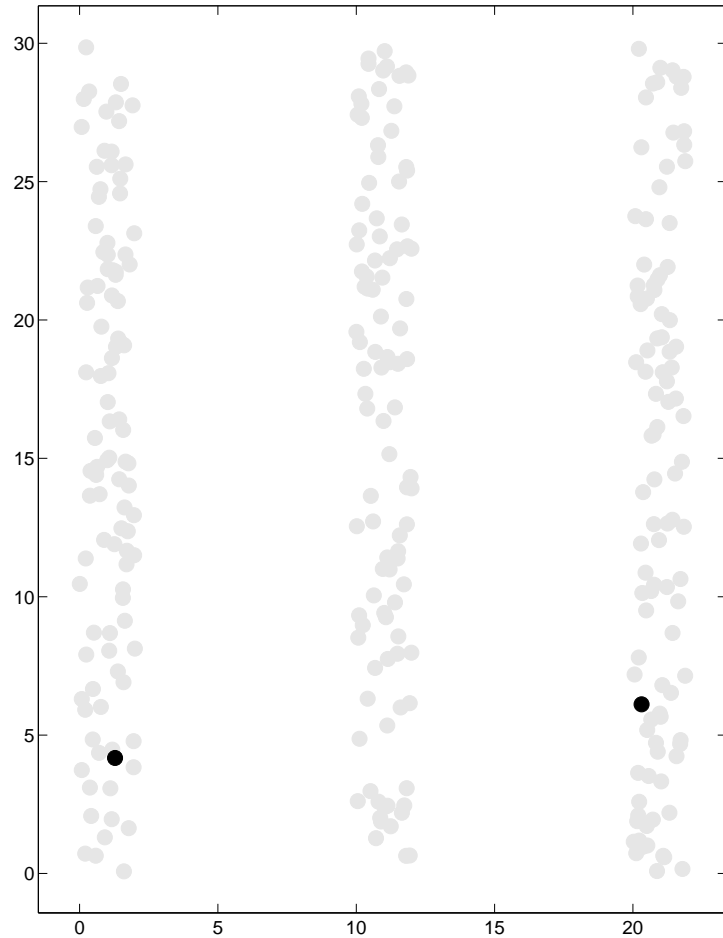
Segmentation with object knowledge



Papers

1. **Grouping with bias**,
Stella X. Yu and Jianbo Shi, NIPS 2001.
2. **Concurrent object recognition and segmentation by graph partitioning**,
Stella X. Yu, Ralph Gross and Jianbo Shi, NIPS 2002.
3. **Object-specific figure-ground segregation**,
Stella X. Yu and Jianbo Shi, submitted to CVPR 2003.
4. Other issues: www.cs.cmu.edu/~xingyu/research.html

Basic problem: grouping with constraints



$$\max \quad \epsilon(f|D)$$

$$\text{s.t.} \quad f(i) = f(j)$$

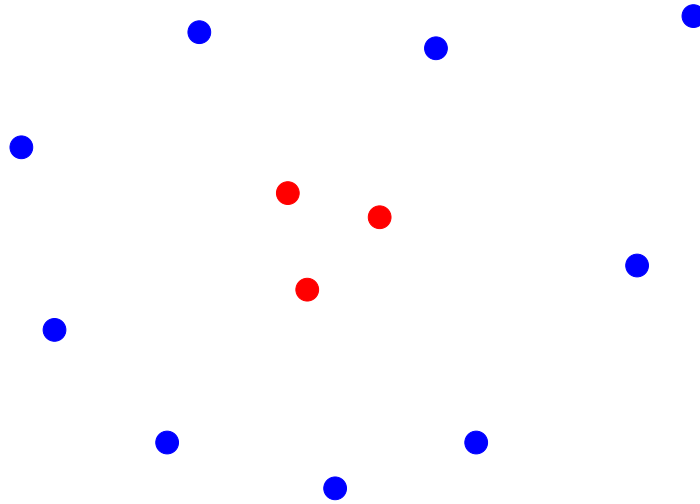
ϵ : goodness of grouping

f : label distribution

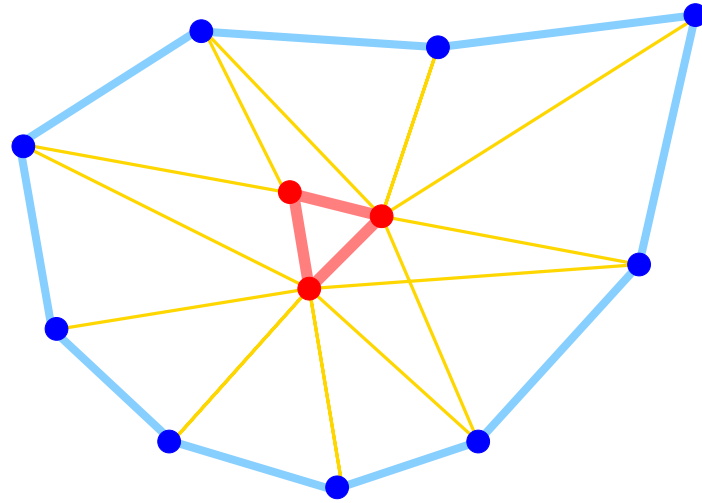
D : observation, e.g. data location

i, j : data index

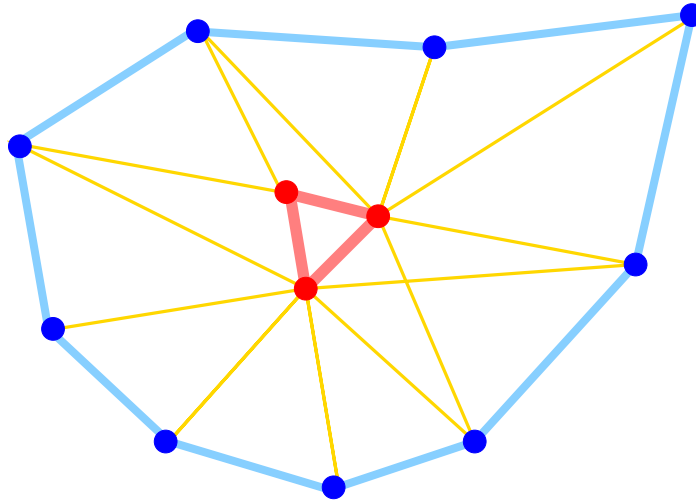
Review on normalized cuts



Review on normalized cuts



Review on normalized cuts



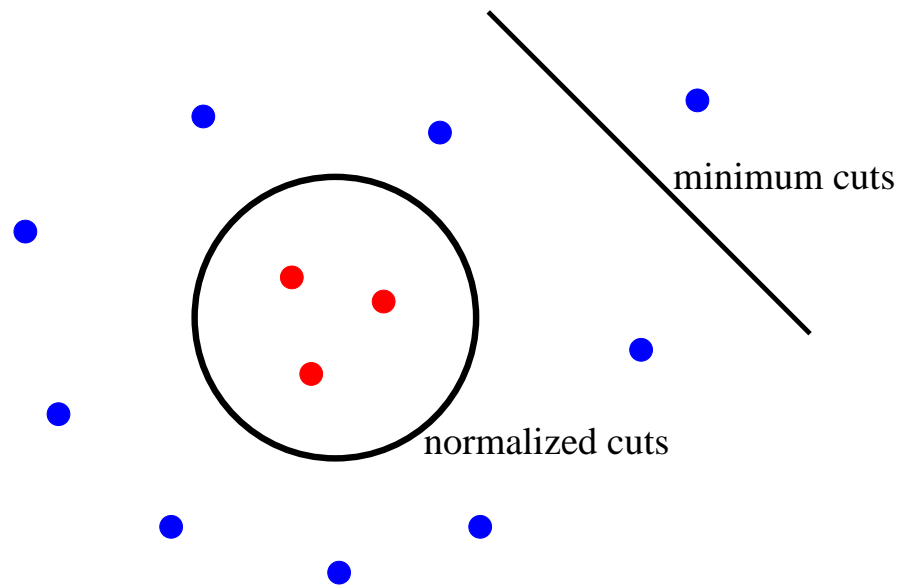
$$\begin{aligned}\epsilon(f|D) &= \sum \frac{\text{within-group weights}}{\text{group degree}} \\ &= \frac{\text{red}}{\text{red} + \text{yellow}} + \frac{\text{blue}}{\text{blue} + \text{yellow}}\end{aligned}$$

Properties of normalized cuts

1. Duality

max within-group weight ratio \leftrightarrow
min between-group weight ratio

2. Normalization for global structure



Computing constrained normalized cuts

Optimization problem:

$$\begin{aligned} &\text{maximize} && \epsilon(z; W) = \frac{z^T W z}{z^T D z} \\ &\text{subject to} && U^T z = 0. \end{aligned}$$

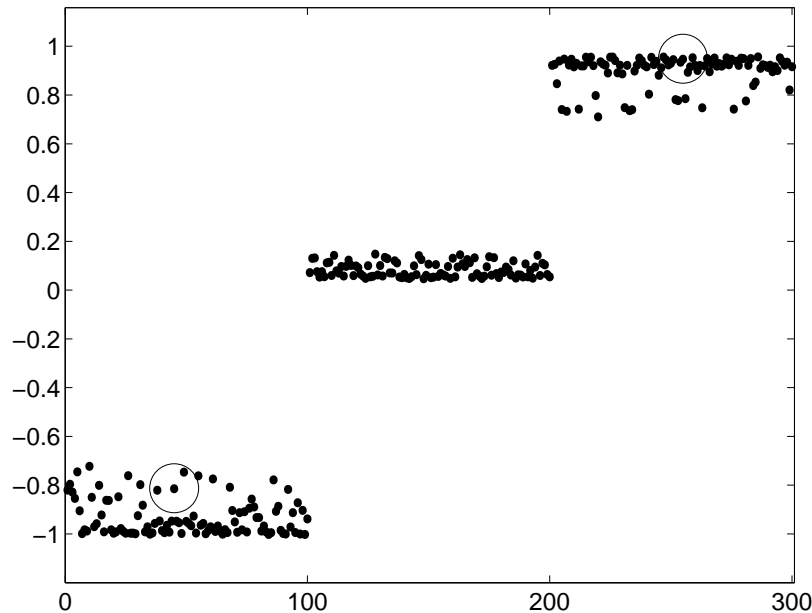
Solution: eigenvector with the largest nontrivial eigenvalue:

$$\begin{aligned} QPz^* &= \lambda z^*, \\ P &= D^{-1}W, \end{aligned}$$

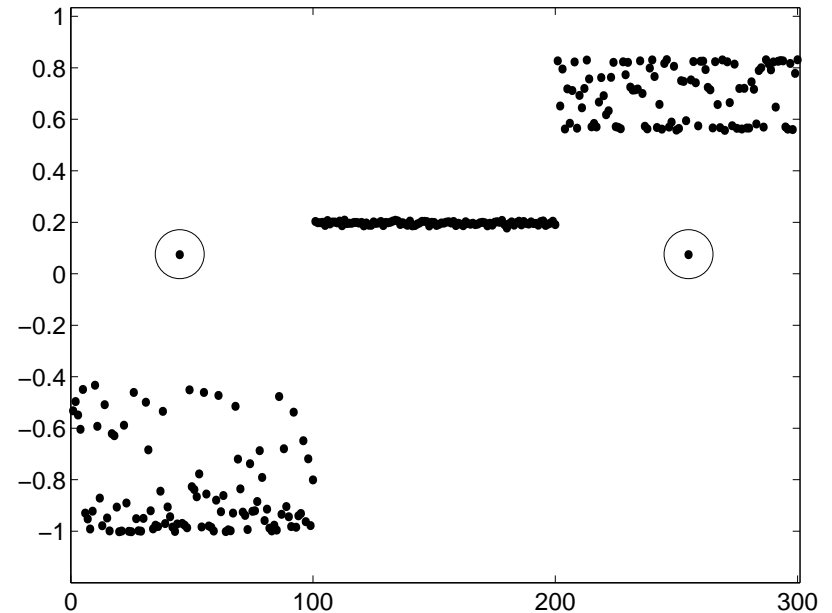
where Q is a projector to the feasible solution space:

$$Q = I - D^{-1}U(U^T D^{-1}U)^{-1}U^T.$$

Results given partial grouping constraints



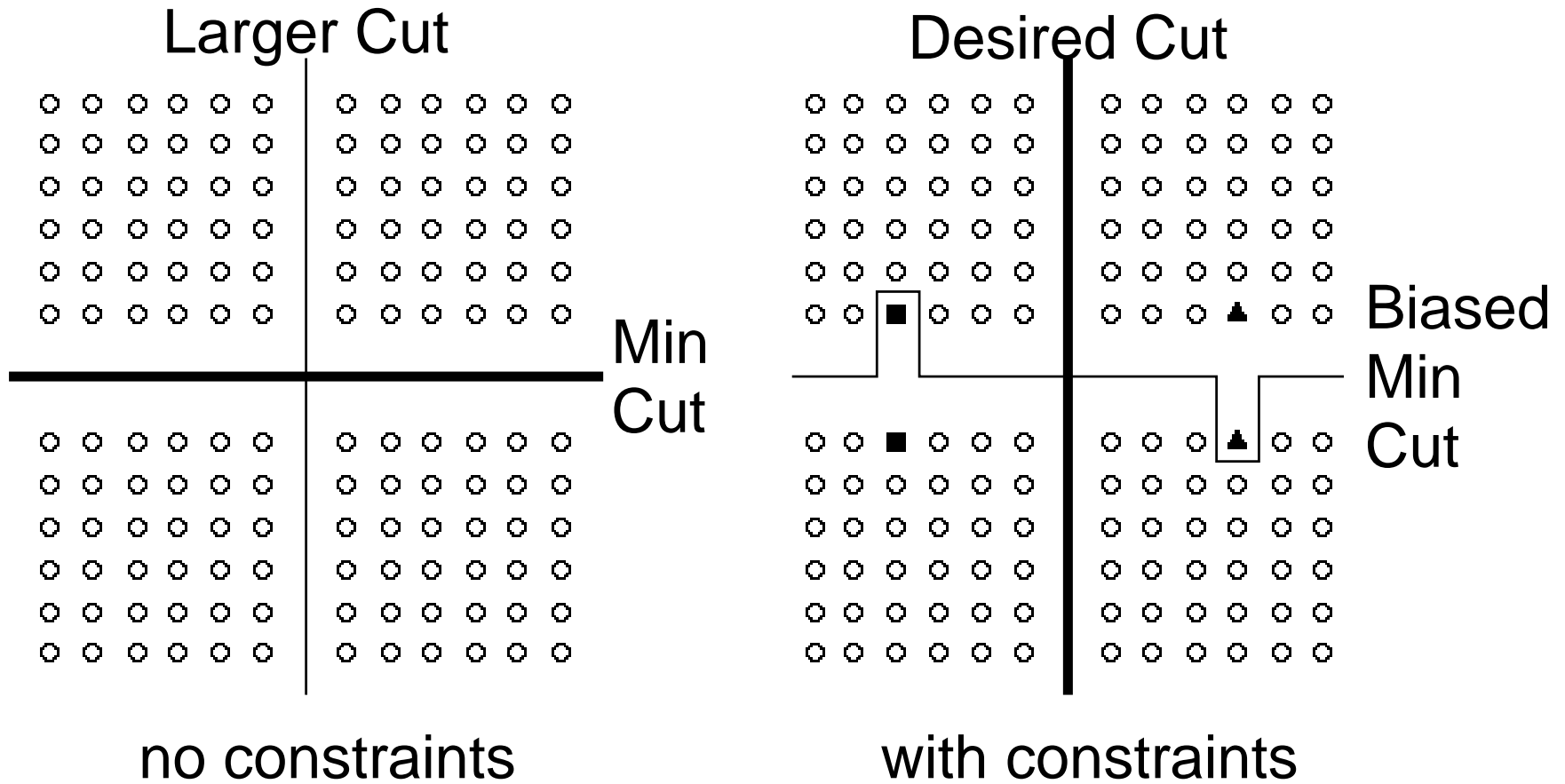
natural clustering



constrained clustering

Not desirable when partial grouping cues are **sparse**!

Why simple constraints are not sufficient



Solution: condition a grouping

$$\max \quad \epsilon(f|D)$$

$$\text{s.t.} \quad S \circ f(i) = S \circ f(j)$$

S : local smoothing kernel

$S \circ f$: average label distribution

Solution: condition a grouping

$$\begin{aligned} \max \quad & \epsilon(f|D) \\ \text{s.t.} \quad & S \circ f(i) = S \circ f(j) \end{aligned}$$

S : local smoothing kernel

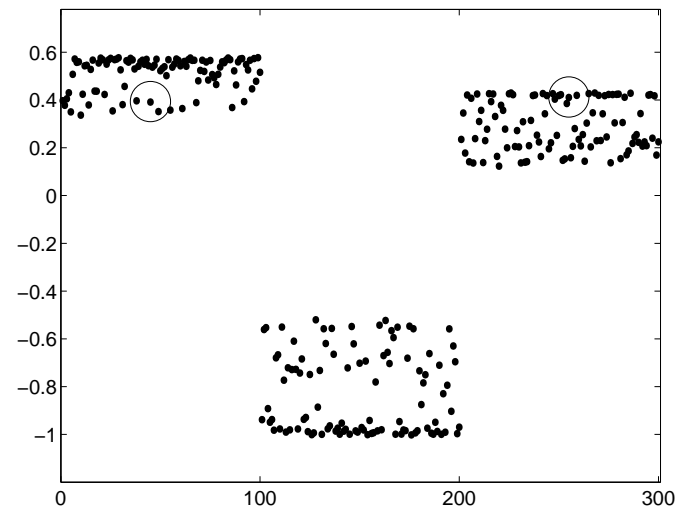
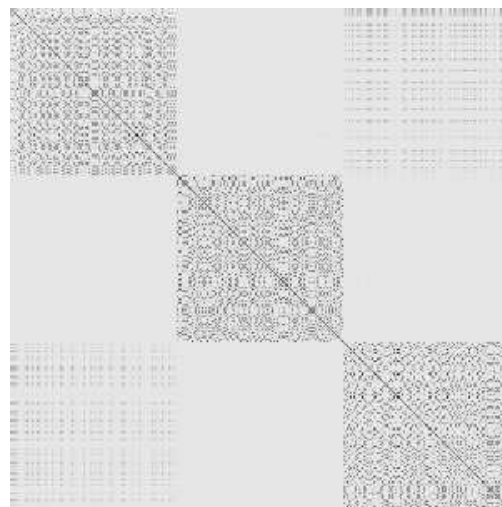
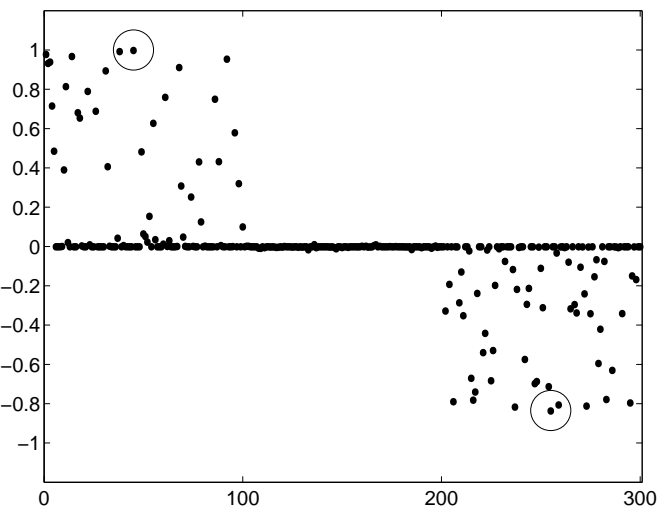
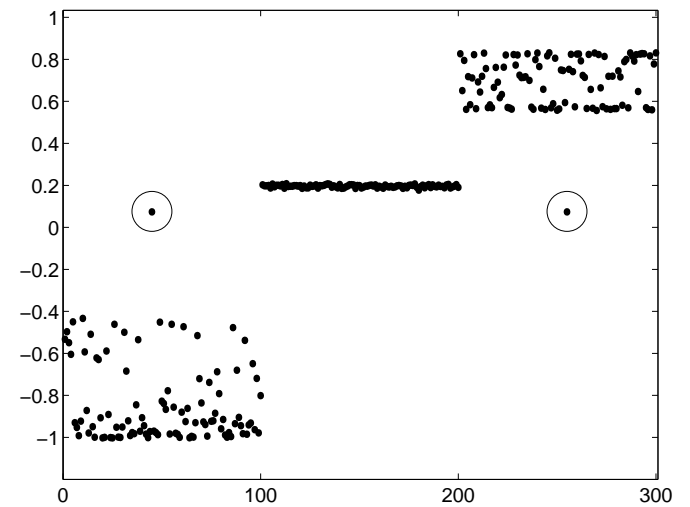
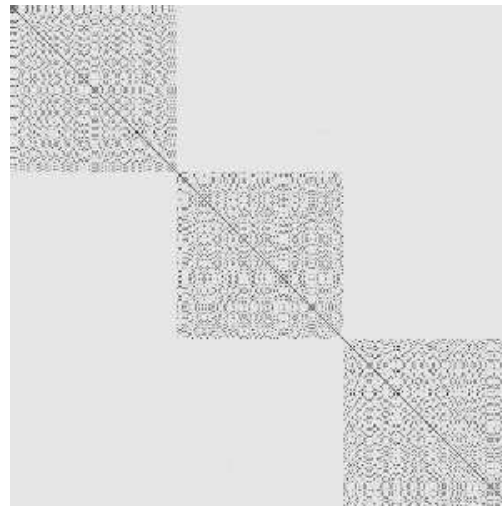
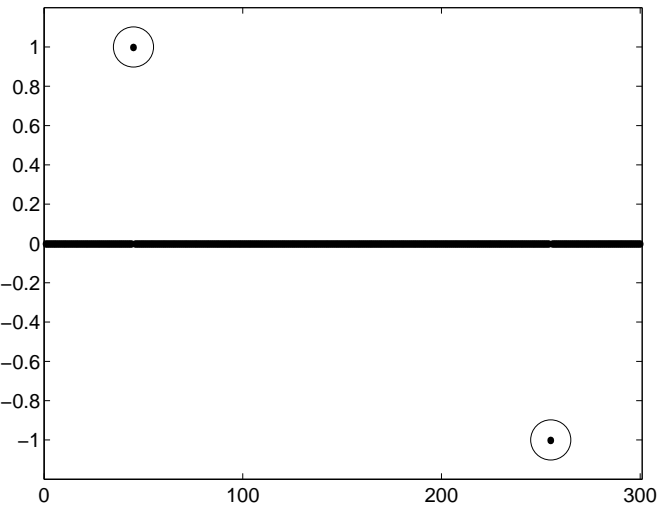
$S \circ f$: average label distribution

$$\begin{aligned} \max \quad & \epsilon(z; W) = \frac{z^T W z}{z^T D z} \\ \text{s.t.} \quad & (P^T U)^T z = 0 \end{aligned}$$

P : transition matrix

$$\sum_k P(i, k) z(k) = \sum_k P(j, k) z(k)$$

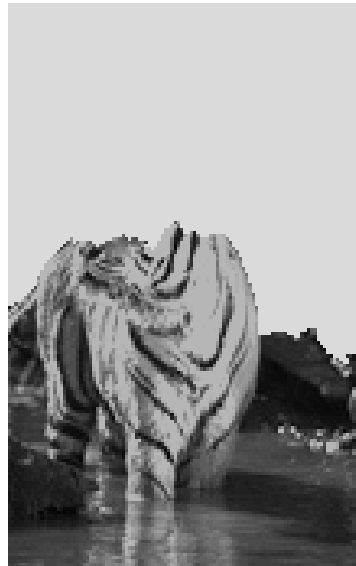
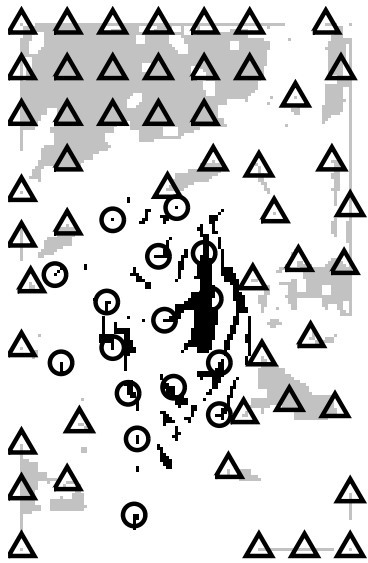
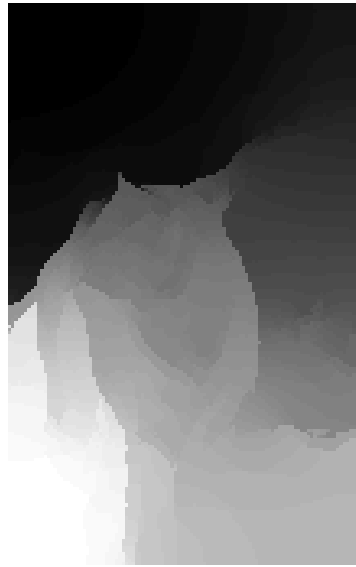
Results given smoothed constraints



constraints

QP

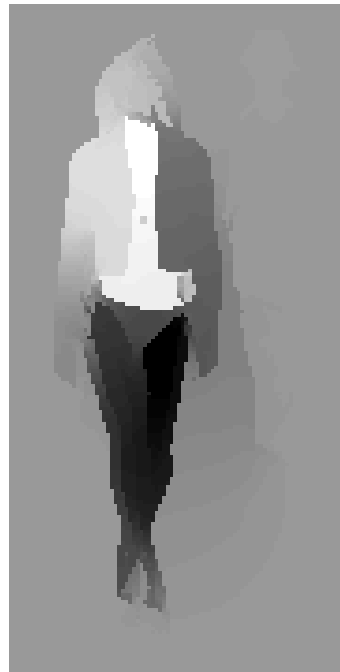
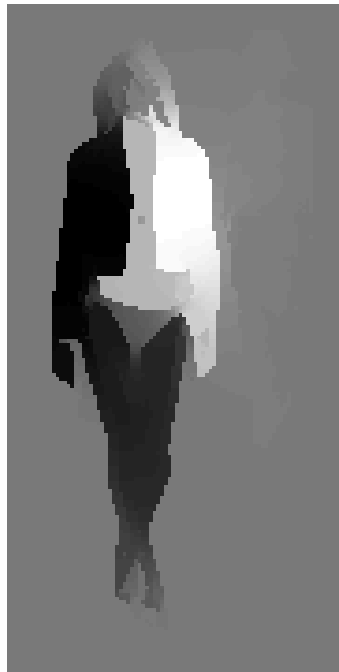
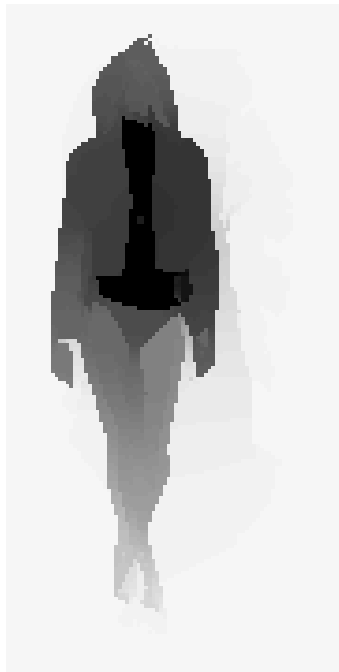
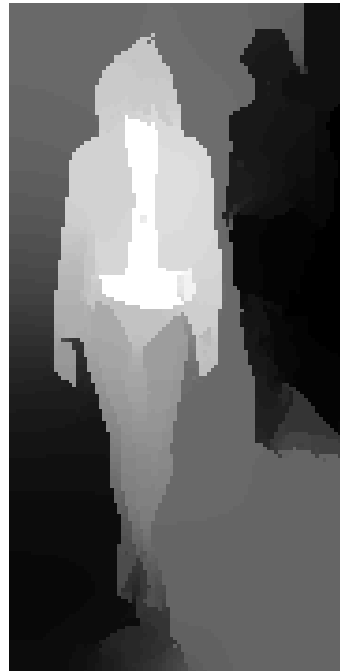
eigenvector



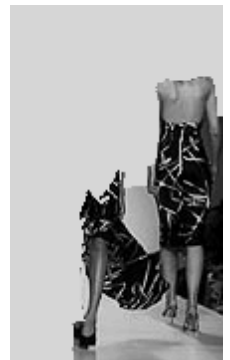
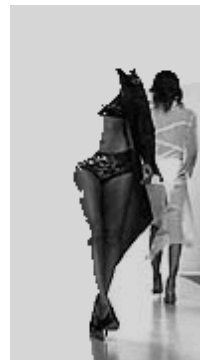
173 × 109

No constraints: 30s

Smoothed constraints: 24s



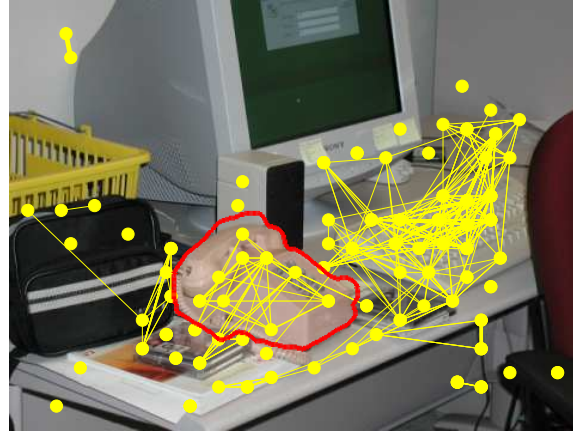
Segment an object without knowing what it is



Approach to object segmentation



Patches



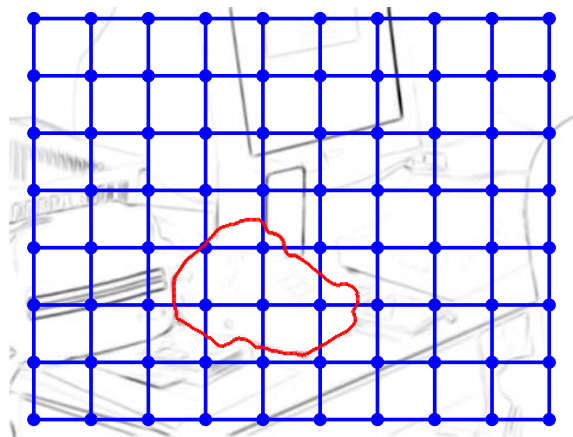
Patch grouping



Correspondence



Edges

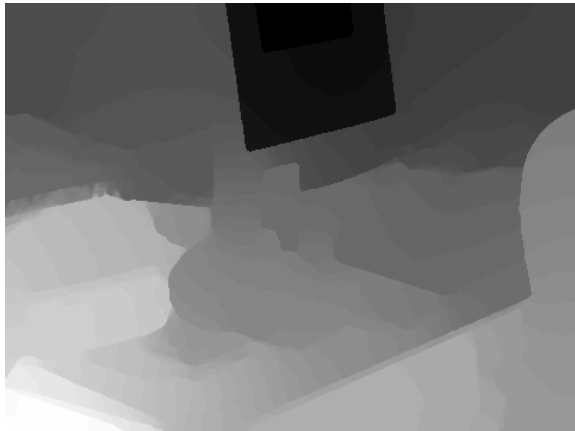


Pixel grouping

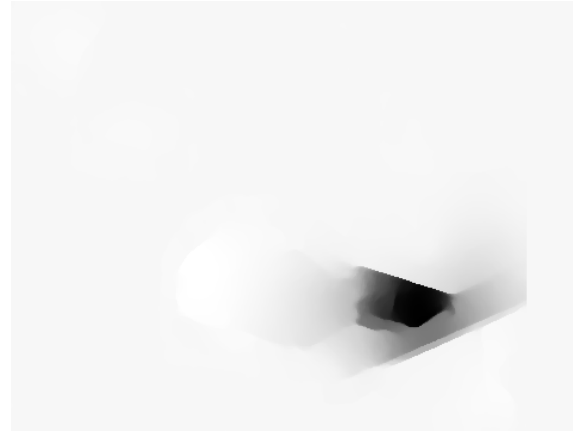


Figure-ground

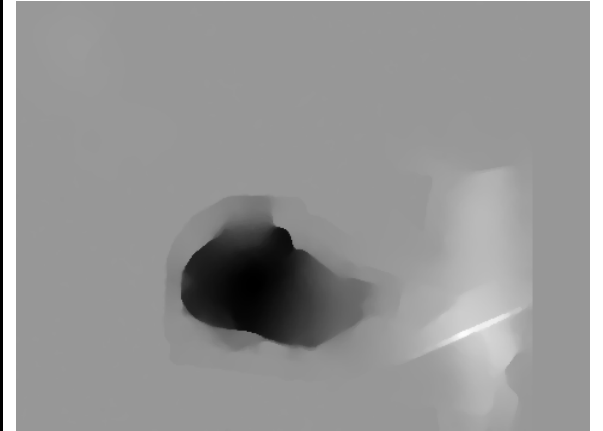
How object knowledge helps



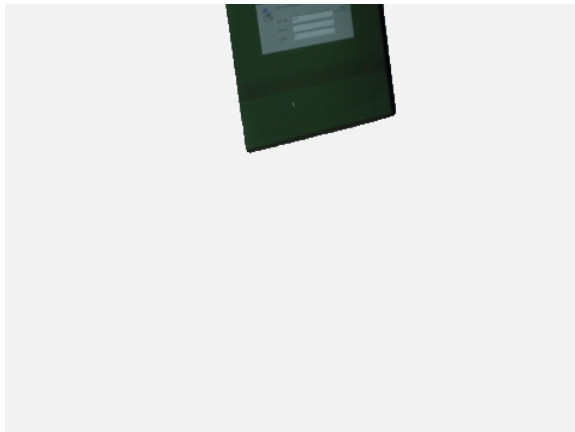
541s



150s



206s



Pixel only



Pixel w/ ROI



Pixel-patch

How low-level image features help



patches



correspondence



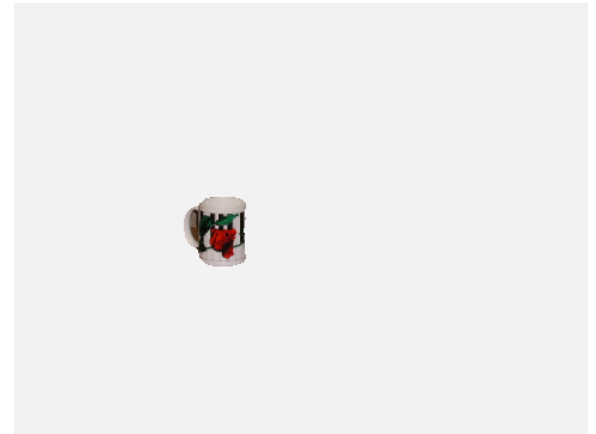
image



edges



eigenvector



segmentation